Giant Magnons[1]

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Integrability in Gauge and String Theory.
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version date: July 25, 2006

[1]hep-th/0604135

Contents

1. Introduction

1.1. String Theory and AdS/CFT correspondence

- Gauge Theory/String Theory duality. Can we build a strong coupling description of QCD out of gravity?
- String Theory on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N} = 4$ SYM
- Important Objects: generating functionals, partition functions, weight/energy spectrum...
- Problems and Solutions. Weak/Strong duality. BPS objects.

1.2. Spin Chains, BMN and Semiclassical Strings

- Following that idea, we consider the BPS state $Tr[Z^J]$. This corresponds to a massless mode of 10d SUGRA propagating on the sphere. $\Delta - J = 0$.
- What to do next? Smart idea: Consider $Tr[WZ^J] = J_{1+i2,5+i6}Tr[Z^J]$. This is the state with lowest energy and charge $J' = 1$ over the vacuum. Thus, it is BPS.

- Now we are getting somewhere. We have an impurity on a chain of letters. Let us just do what we always do: Let's construct a Bloch State.

$$O_p \sim \sum_l e^{ilp}(\cdots ZZZWZZZ\cdots) \qquad (1)$$

- There's a catch! An honest string state should contain, at least, two excitations with opposite momenta to be non vanishing because of trace. These states are not, however, BPS. They are the next best thing to BPS states: Impurities running in the chain. This problem can now be mapped to a Spin Chain with $\Delta \to H$ where our fundamental blocks are the magnons we just described. [Minahan and Zarembo; Beisert, ...]

- We will forget about the trace and consider single excitation states at infinite J. Are they BPS?

- It turns out that in the $J \to \infty$ limit, the energy of these states can be calculated in perturbation theory with the effective coupling $\lambda' = \frac{\lambda}{J^2}$. Using symmetry arguments it is possible to obtain the exact formula:

$$E - J = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} \qquad (2)$$

[Beisert]

This result uses the $su(2|2) \times su(2|2)$ symmetry algebra extended by central charges that make the chain dynamic. Closed string states are neutral under these charges. Symmetries do not constrain $f(\lambda) \to f(\lambda) = \frac{\lambda}{\pi^2}$ experimental result!

- On the string theory size this corresponds to states in the Plane Wave (BMN) limit: an exact quantization at $J, \lambda \to \infty$ and $pJ =$ fixed is possible [Berenstein, Maldacena, Nastase]. $\lambda' = \frac{\lambda}{J^2}$ expansion.

$$E - J = \sqrt{1 + \frac{\lambda n^2}{J^2}} \quad p = \frac{2\pi n}{J} \tag{3}$$

- Other tools: Semiclassical Strings. Large quantum numbers. [GPK; Tseytlin (and refs. therein)...]

1.3. Idea of this work

The aim of this work is:

- to reproduce the full structure of the magnon spectrum (sine dependence) from the string theory side. We will be working in the large radius limit. This means $\lambda \to \infty$. Then, we expect

$$E - J = \frac{\sqrt{\lambda}}{\pi} \left| \sin \frac{p}{2} \right| \tag{4}$$

Note that the limit we are considering here is different from BMN: p is fixed and λ interpolates between gauge theory and string theory results. We will also argue that the symmetries responsible for this formula in the string theory side are the same as the ones studied by [Beisert].

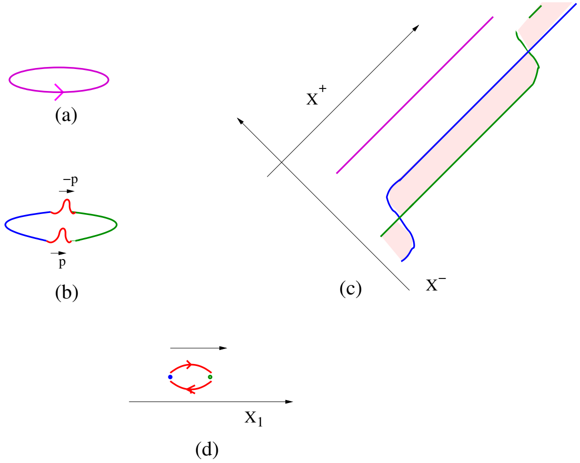
- Identify string states dual to magnon states.
- Using these solutions, compute the semiclassical S-matrix dressing factor and compare with AFS formula. [Arutyunov, Frolov, Staudacher]
- Bound States?

2. Giant Magnons in Semiclassical String Theory

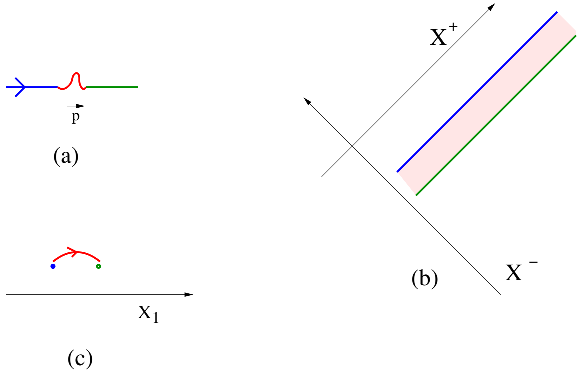
2.1. Localized excitations in a string

Strings in Flat Space

- Strings of finite worldsheet length.



- Strings of infinite worldsheet length.

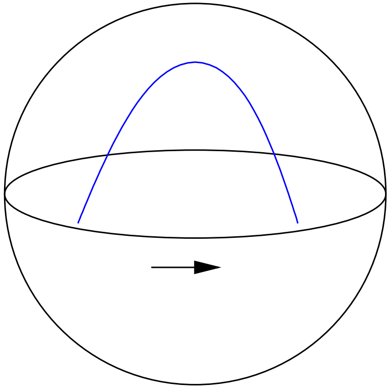


2.2. Giant Magnons in the large J limit

- Solution obtained by minimizing the string action.

$$\cos\theta = \frac{\cos\theta_0}{\cosh\left[\frac{x-\sin\theta_0 t}{\cos\theta_0}\right]} = \frac{\sin\frac{p}{2}}{\cosh\left[\frac{x-\cos\frac{p}{2}t}{\sin\frac{p}{2}}\right]} \tag{5}$$

$$\tan(\varphi - t) = \cot\theta_0 \, \tanh\left[\frac{x-\sin\theta_0 t}{\cos\theta_0}\right] = \tan\frac{p}{2}\, \tanh\left[\frac{x-\cos\frac{p}{2}t}{\sin\frac{p}{2}}\right] \tag{6}$$



- Coordinates and momentum identified by

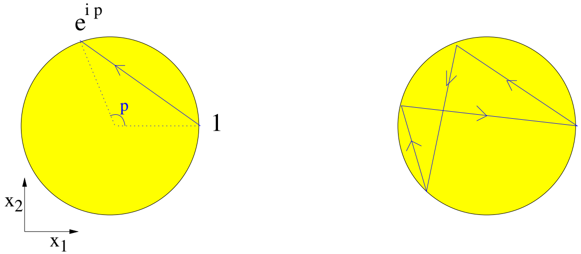
$$\frac{dJ}{dx} = \frac{\sqrt{\lambda}}{2\pi} \qquad \text{or} \qquad dl = \frac{\sqrt{\lambda}}{2\pi} dx \tag{7}$$

$$v_{gauge} = \frac{dl}{dt} = \frac{d\epsilon(p)}{dp} = \frac{\sqrt{\lambda}}{2\pi} \cos\frac{p}{2}\,, \qquad \text{for} \quad p > 0 \tag{8}$$

$$v_{string} = \frac{dx}{dt} = \sin\theta_0 = \cos\frac{\Delta\varphi}{2} \tag{9}$$

- Note that we allowed for an open string solution. This is equivalent to relaxing the zero mode of the Virasoro constraints (or the trace in the gauge theory). We can do this for an infinite string.

2.3. LLM coordinates, BPS states and SUSY algebra



(a) (b)

- This coordinates are just a projection of the sphere onto the plane.
- The SUSY algebra in 10d SUGRA contains gauge transformation of the B field under which the string is charged [Schwarz]. Therefore, open stretched strings can carry winding charges associated. It is these charges that make the state BPS. Analogous to strings stretched between branes.

- This charges are proportional to the string length. Therefore:

$$k^1 + ik^2 = \frac{R^2}{2\pi\alpha'}(e^{ip} - 1) = i\frac{\sqrt{\lambda}}{\pi}e^{i\frac{p}{2}}\sin\frac{p}{2} \tag{10}$$

$$E - J = k^0 = \sqrt{1 + |k_1 + ik_2|^2} = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} \tag{11}$$

- This algebra looks like the usual 2+1 Poincare algebra. Notice, however, that boosts are not a symmetry but an outer automorphism of the algebra.
- Finally: Closed string states are neutral. (therefore, not BPS in general).

3. S-matrix, Bound States and the Sine-Gordon Model

3.1. Sine-Gordon solutions, time delays and the S-Matrix

- Classical correspondence between Sine-Gordon and strings on $R \times S^2$.

- Sine-Gordon solitons $\longleftrightarrow$ String Theory Magnons.
- Nontrivial energy relation $\epsilon = \frac{\sqrt{\lambda}}{\pi}\frac{1}{E_{sg}}$.
- Lorentz symmetry in Sine-Gordon responsible for one parameter solutions in string theory. Again, this is not a symmetry of the theory.
- Scattering solutions in sine-gordon map to scattering of magnons.
- x and t are the same $\longrightarrow$ time delays are the same (S-matrices are not though).
- WKB approximation gives phase as $\frac{\partial\delta_{12}(\epsilon_1,\epsilon_2)}{\partial\epsilon_1} = \Delta T_{12}$ [Jackiw, Woo]. This agrees (modulo length redefinition) with the AFS formula, derived in an indirect way.

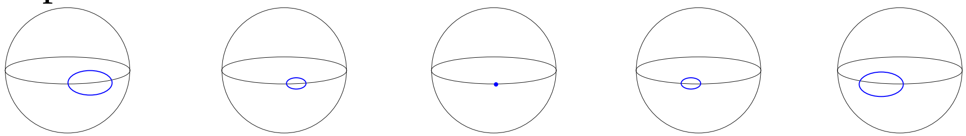
$$\Delta T_{12} = \frac{2}{\gamma_1 v_1}\log v_{cm} \tag{12}$$

$$\delta = \frac{\sqrt{\lambda}}{\pi}\left\{\left[-\cos\frac{p_1}{2}+\cos\frac{p_2}{2}\right]\log\left[\frac{1-\cos\frac{p_1-p_2}{2}}{1-\cos\frac{p_1+p_2}{2}}\right]\right\} - p_1\frac{\sqrt{\lambda}}{\pi}\sin\frac{p_2}{2} \tag{13}$$

- Length redefinition. Far away from the excitation E and J have constant densities. But inside the excitation, our gauge says the have just constant E. To compare with gauge theory we should go to coordinates where J is constant. The mismatch will be given in the phase as $e^{ip\Delta\text{Length}}$. (Compare with BA equations).

$$\Delta l = \int dx \frac{dJ}{dx} = \int dx \frac{dE}{dx} - (\frac{dE}{dx} - \frac{dJ}{dx}) = \frac{2\pi}{\sqrt{\lambda}} \Delta x - \epsilon \tag{14}$$

- Space-time picture.



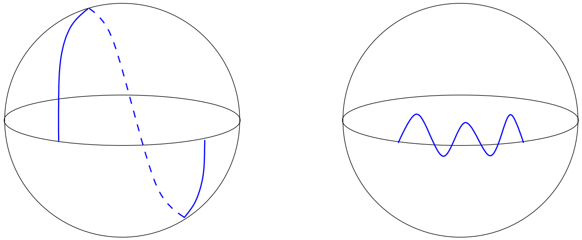
3.2. Bound States

- Bound State solutions are given by breathers.
- Energy given by sum of energies (integrability) with complex momenta ($p_i = p \pm iq$). Inversion and Numerical studies.

- Time dependence allows for WKB quantization. $dn = \frac{T}{2\pi} d\epsilon|_p$.

$$\epsilon = \frac{\sqrt{\lambda}}{\pi}(\sin\frac{p_1}{2} + \sin\frac{p_2}{2}) = \frac{2\sqrt{\lambda}}{\pi}\sin\frac{p}{2}\cosh\frac{q}{2} = \sqrt{n^2 + \frac{4\lambda}{\pi^2}\sin^2\frac{p}{2}} \quad (15)$$

- Space-time picture.



(a) (b)

- Explicit bound state solutions: Backlund transformations and the dressing method [Spradlin and Volovich]
- Stability and Poles.

4. Further progress

- BPS bound states and *the one under the square root formula* [Dorey; Chen, Dorey and Okamura]

- Finite Size Effects [Arutyunov, Frolov, Zamaklar]
- Analysis of classical solutions in similar limits [Minahan, Tirziu and Tseytlin; Kruczenski, Russo and Tseytlin]
- Magnons in β-deformed backgrounds [Chong-Sun Chu, Georgiou and Khoze]
- Multispin solutions [Spradlin adn Volovich; Bobev and Rashkov; Kruczenski, Russo and Tseytlin]

5. Conclusions

- Fundamental excitations have been identified on both sides of correspondence.
- Symmetries responsible for the spectrum were matched.
- S-matrix calculated semiclassically and found to agree with the AFS formula.
- Bound state spectrum. Coupling dependence. Poles in S-Matrix?
- Future directions: further investigation of the S-matrix for more general bound states (in progress). Crossing symmetry, poles, etc.