Magnons in gauge and string theory

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Outline

- Introductory Remarks
- Perturbation theory in *N=4 SYM* and its relation to spin chains
- Magnons in gauge theory ($SU(2)$ sector)
- Magnons in string theory

Why $\mathcal{N}$=4 Super Yang-Mills?

- It is not QCD (No running of coupling, confinement etc.)
- AdS/CFT □ Use string theory to study gauge theory
- We can make it QCD-like by breaking supersymmetries
- It might be "solvable"

Can N =4 Super Yang-Mills be solved?

- Large N limit only
- If so we can take λ~ 1 (improve comparison to QCD) (λ ‘t Hooft coupling: $\lambda = g^2 N$)
- First step: find the spectrum
- Relate the theory to a spin chain
- Integrability □ finding the S-matrix
- Magnons □ Only need to know the poles and zeros of the S-matrix

A few facts about $N=4$

- Conformal □ All info is contained in correlation functions of gauge invariant ops.

- Large N single trace ops.

- 6 adjoint scalar fields

- $SU(2)$ sector: closed in pert. theory
  $Z=\varphi_1 + i\,\varphi_2$, $W=\varphi_3 + i\,\varphi_4$

Gauge/String Correspondence

Planar Limit:

$$\mathcal{O} = \mathrm{Tr}[ZZZ...WW..ZZZ..] \Leftrightarrow \text{String state}$$

$$\Delta = E$$

$SU(2)$ sector: J Ws, J Zs

Many Successes: BMN, Long wavelength (2 loops)

Many puzzles: 3loop mismatch, 2 loop agreement

Finite size effects and comparing small λ with large λ

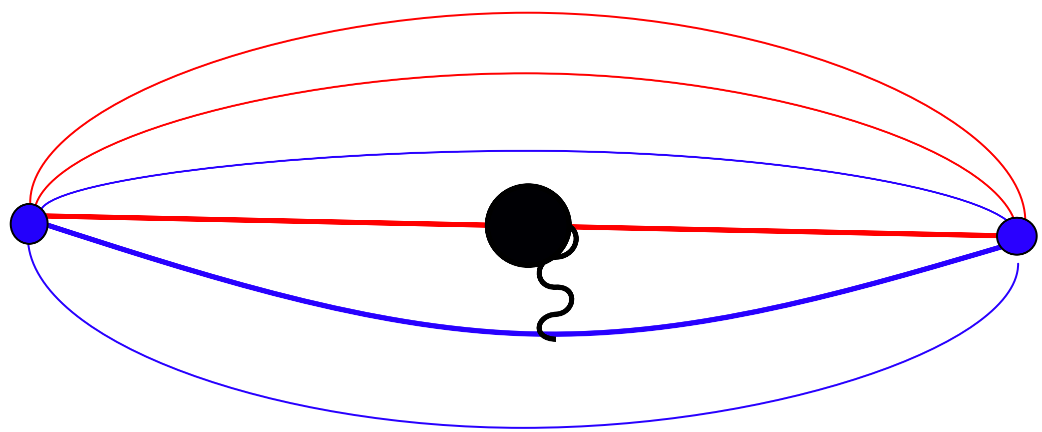
$SU(2)$ Sector of N=4

$O_L(x) = \mathrm{Tr}\,(\,ZZZ\ldots WW\ldots ZW\ldots WWZW\,)$

$Z =)_1 + i\,)_2$
$W =)_3 + i\,)_4$

$\otimes_0 = L$

Tree level:
One loop:



These two diagrams don’t mix the flavor

SU(2) Sector of *N*=4

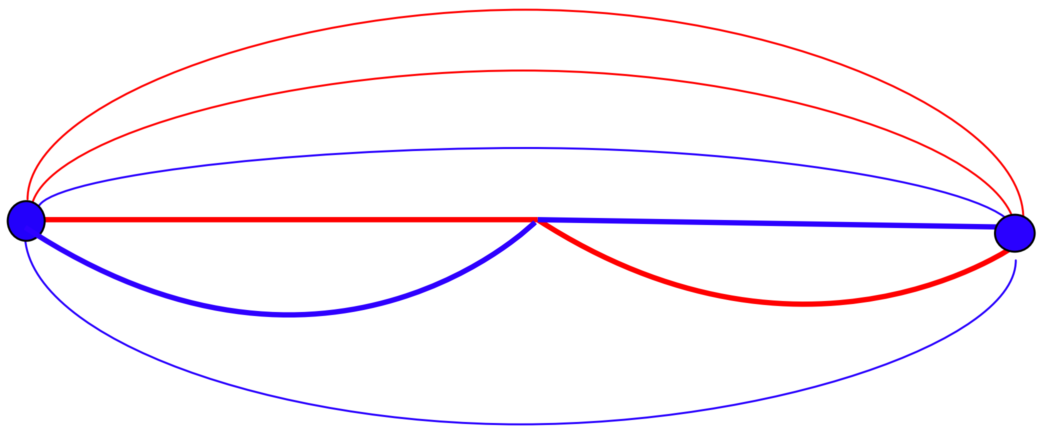
$O_L(x) = \mathrm{Tr}\,(\,ZZZ\ldots WW\ldots ZW\ldots WWZW\,)$

$Z =)_1 + i\,)_2$
$W =)_3 + i\,)_4$

$\otimes_0 = L$

Operator Mixing

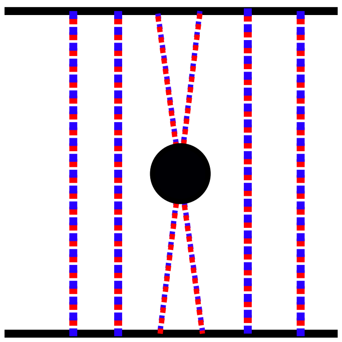
One loop:



$O_L(x) = \mathrm{Tr}\,(\,ZZZ\ldots WW\ldots WZ\ldots WWZW\,)$

One Loop

SU(2) sector



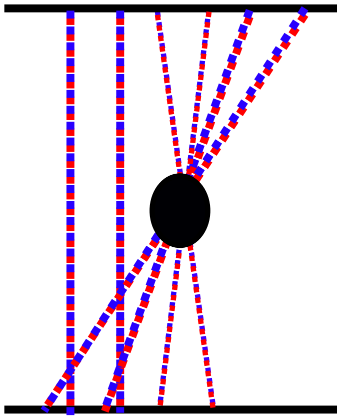
Planar:

$$H_2 = \frac{\lambda}{8\pi^2} \sum_{j=1}^{L} (1 - P_{j,j+1})$$

K. Zarembo, JM 2002

$$g^2 = \frac{\lambda}{8\pi^2}$$

Higher Loops



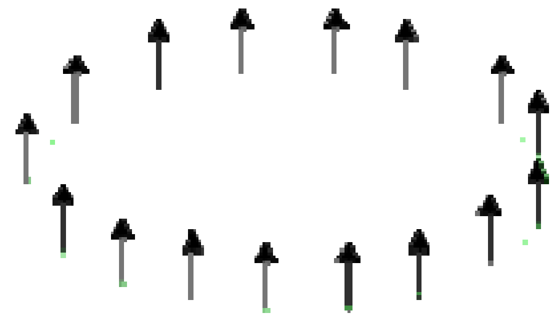
$$H = \sum_{n=1}^{\infty} g^{2n} H_{2n}$$

H_{2n} ranges over n sites

3-loops: Beisert, Kristjansen & Staudacher
5-loops: Beisert
Eden Jarczak & Sokatchev

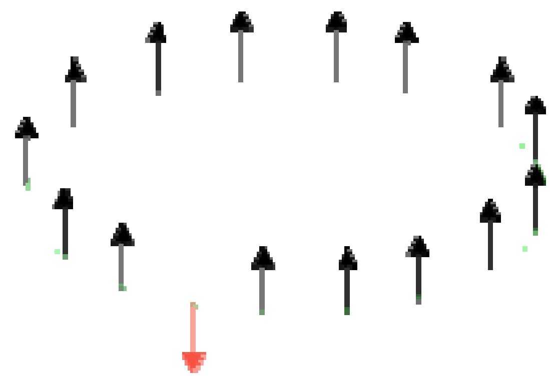
The Bethe Ansatz

Ground state:



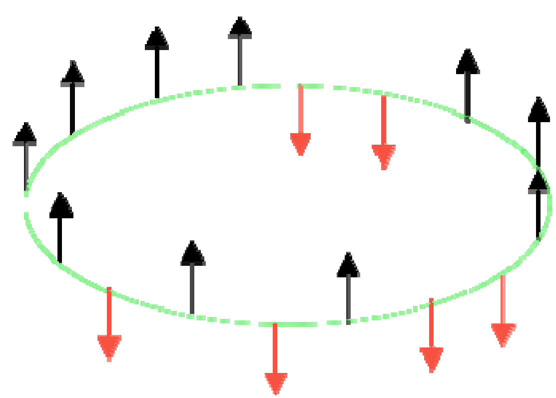
$$|0\rangle$$

One magnon:



$$|p\rangle = \frac{1}{\sqrt{L}}\sum_{\ell=1}^{L} e^{ip\ell}|\ell\rangle$$

J_1 magnons:



$$\sum_{k=1}^{J_1} p_k = 2\pi n$$

$$|p_1 \ldots p_{J_1}\rangle \neq \frac{1}{L^{J_1/2}}\prod_{j=1}^{J_1}\sum_{\ell_j=1}^{L} e^{ip_j\ell_j}|\ell_1\ldots\ell_{J_1}\rangle$$

Integrability: All scattering is reduced to two body

S-matrix: $S(p_j, p_k)$

Quantization: $e^{ip_j L} = \prod_{k \neq j}^{J_1} S^{-1}(p_j, p_k)$ $L = J_1 + J_2$

One-loop: $\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j}^{J_1} \frac{u_j - u_k + i}{u_j - u_k - i}$

Bethe

All loops: $\left(\frac{x(u_j + i/2)}{x(u_j - i/2)}\right)^L = \prod_{k \neq j}^{J_1} \frac{u_j - u_k + i}{u_j - u_k - i}$

(asymptotic only)

Beisert,Dipple,Staudacher

$$x(u) = \frac{1}{2}\left(u + \sqrt{u^2 - 2g^2}\right)$$

Magnon dispersion:

$$c(p_j) = ig^2 \left(\frac{1}{x(u_j + i/2)} - \frac{1}{x(u_j - i/2)} \right)$$

$$E = L + ig^2 \sum_{j=1}^{J_1} \left(\frac{1}{x(u_j + i/2)} - \frac{1}{x(u_j - i/2)} \right)$$

$$u_j = \frac{1}{2} \cot \frac{p_j}{2} \sqrt{1 + 8g^2 \sin^2 \frac{p_j}{2}}$$

$$E = J_2 + \sum_{j=1}^{J_1} \sqrt{1 + 8g^2 \sin^2 \frac{p_j}{2}}$$

Beisert Dippel
& Staudacher
Beisert

Trace condition: $$\sum_j p_j = 2\pi n$$

Bethe Strings

$$e^{ip_jL} = \prod_{k\neq j}^{J_1} \frac{u_j - u_k + i}{u_j - u_k - i}$$

$$L \to \infty$$

If p_j has an imaginary part: lhs $\Rightarrow$ 0 or ∞

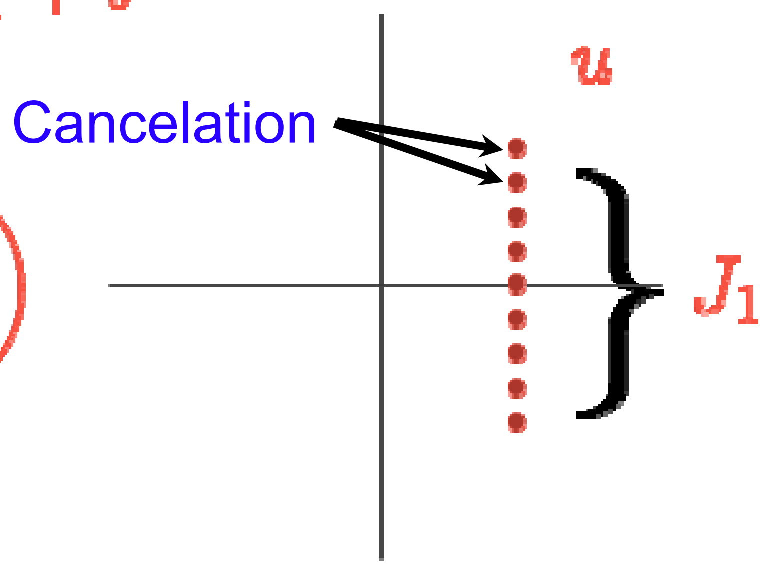
To compensate: $u_j = u_{j+1} + i$

$$\epsilon(p_j) = ig^2 \left(\frac{1}{x(u_j + i/2)} - \frac{1}{x(u_j - i/2)} \right)$$

$$E = L + ig^2 \left(\frac{1}{x(u_1 + i/2)} - \frac{1}{x(u_{J_1} - i/2)} \right)$$

$$= J_2 + \sqrt{J_1^2 + 8g^2 \sin^2 \frac{p}{2}}$$

Dorey



Long Bethe strings are solitons

If J >> g, J >>1 Heisenberg

Classical Limit Landau-Lifschitz equation:

$$\dot{\vec{S}} = -2g^2 \vec{S} \times \vec{S}''$$

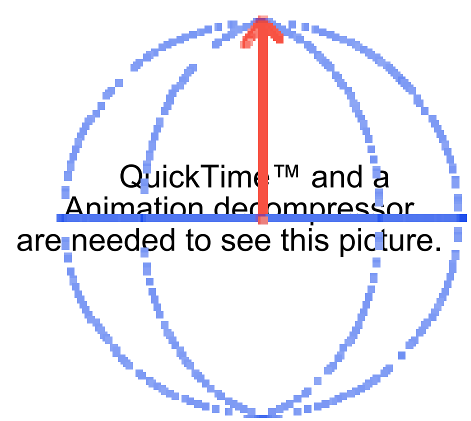
$$\varphi = -\frac{E}{J_1} - \frac{\sin p}{J_1}(x - vt) - \arctan\left[\tan\frac{p}{2}\tanh\left(\frac{x - vt}{\Gamma}\right)\right]$$

$$\sin\frac{\theta}{2} = \sin\frac{p}{2}\,\mathrm{sech}\left(\frac{x - vt}{\Gamma}\right)$$

$$\Gamma = \frac{J_1}{2\sin^2\frac{p}{2}}$$

$$v = \frac{2g^2}{J_1}\sin p$$

Classical Spin
(at one site):



Lakshmanan (1977)
Takhtajan (1977)
Fogedby (1980)

Comparison to Strings

Hofman & Maldacena
Dorey

Chen, Dorey & Okamura
Arutyunov, Frolov & Zamaklar
Tirziu, Tseytlin & JM
Spradlin & Volovich
Kruczenski, Russo & Tseytlin

Restrict to $R \times S^3$

SU(2) x SU(2) symmetry

$$ds^2 = -dt^2 + \cos^2\theta \, d\varphi_1^2 + d\theta^2 + \sin^2\theta \, d\varphi_2^2$$

Ansatz: $t = \tau \,, \quad \theta = \theta(\sigma) \,, \quad \varphi_1 = w(t - \psi(\sigma)) \,, \quad \varphi_2 = t + \varphi(\sigma)$

Nambu-Goto: $S = \int d\tau d\sigma \sqrt{-\det G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu}$

EOM: $\partial_\sigma \psi - \tan^2\theta \, \partial_\sigma \varphi$

$$\mathcal{L} = \frac{\sqrt{1-w^2}\sqrt{\lambda}}{2\pi} \int d\varphi \, \sqrt{r^2 + r'^2}$$

$r \equiv \sin\theta, \qquad r' \equiv \frac{dr}{d\varphi}$

Generalization of Hofman & Maldacena

Minimize: $r = \frac{\sin\theta_0}{\cos\varphi}$ $\mathcal{L} = \frac{\sqrt{\lambda}}{\pi}\sqrt{1-w^2}\,\sin\frac{p}{2}$ $\sin\frac{p}{2} = \cos\theta_0$

$-\pi/2 + \theta_0 \le \varphi \le \pi/2 - \theta_0$

Conserved Charges: $E,\ J_2$ Infinite

$$E - J_2 = \frac{1}{1-w^2}\mathcal{L}$$

$$J_1 = \frac{w}{1-w^2}\mathcal{L}$$

$$E - J_2 = \sqrt{J_1^2 + 8g^2\sin^2\frac{p}{2}}$$

Same as spin-chain (gauge theory)

nb. $\theta_{\rm LL} = 2\theta_{\rm string},$ $\varphi_{LL} = \varphi_{\rm string} + \psi_{\rm string}$

Giant Magnons in Finite Gap Equations

Monodromy Matrix: $\Omega(x)$ — spectral parameter

unimodularity: $\mathrm{Tr}\,\Omega(x) = 2\cos P(x)$ — quasimomentum

Poles: $P(x) = -\frac{E/4}{x \pm \frac{g}{\sqrt{2}}} - \dots \qquad (x \to \mp \frac{g}{\sqrt{2}})$

Cuts: $P(x+i0) + P(x-i0) = 2\pi n_k, \qquad x \in \mathcal{C}_k$

Condensates:

$P(x) \to P(x) + 2\pi$

Resolvent: $G(x) = P(x) + \frac{E/4}{x + \frac{g}{\sqrt{2}}} + \frac{E/4}{x - \frac{g}{\sqrt{2}}}$

Kazakov, Marshakov, Zarembo & JM

$$G(x) = \sum_k \int_{B_k} dx' \frac{\rho(x')}{x - x'} + \sum_j \int_{C_j} dx' \frac{\rho(x')}{x - x'}$$

Condensate contour

Cut contour

Single magnon only

Relax momentum condition

$$u = x + \frac{g^2}{2x}$$

BDS

Marshakov

$$\int_B du \frac{\rho(u)}{\sqrt{u^2 - 2g^2}} = p$$

$$\int_B du\, \rho(u) = J_1$$

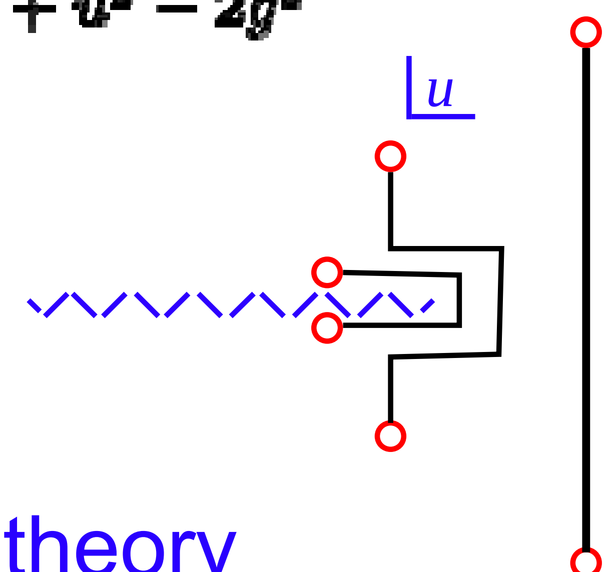
$$2g^2 \int_B du \frac{\rho(u)}{u\sqrt{u^2 - 2g^2} + u^2 - 2g^2} = E - J_2 - J_1$$

⇓

$$\rho(u) = -i$$

Cut in u plane

Decrease J_1 keeping p fixed



Same result as gauge theory

Ende