AdS/CFT and Integrability

- General remarks
- The Pomeron

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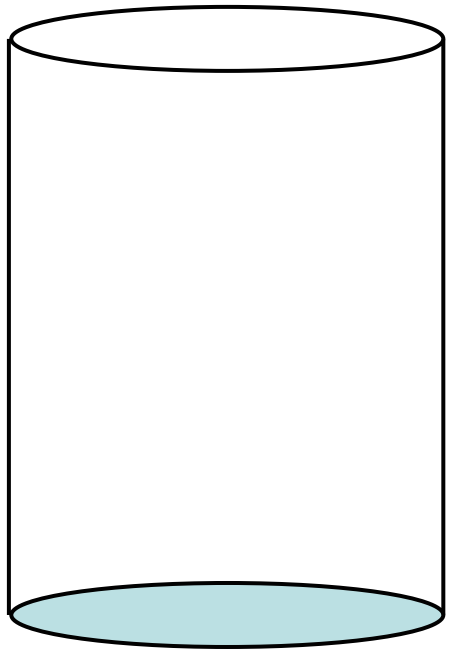
AdS/CFT:

- is a realization of ‘t Hooft’s old conjecture: large-N_c gauge theory at strong coupling = string theory.
- is our most precise *definition* of string theory*. With AdS_5 x S^5 boundary conditions, we can calculate any process in the bulk, e.g.

graviton + graviton at $E >> M_P$
  → black hole
  → final state particles.



*Postscript: Smilga notes that we cannot be sure that 3+1 SYM exists at large λ, in the sense that we could simulate in on the lattice. This is true, but one can resort to 2+1 SYM for a definition-in-principle of string theory: this also has a duality, and is superrenormalizable.

The question *`What is string theory?'* still does not have complete answer: in a holographic theory, changing the boundary conditions is more radical than in a local field theory. Examples:

• AdS_5 x perturbed S^5, AdS_5 x Sasaki-Einstein space are different gauge theories.

• AdS_4 x Calabi-Yau with flux solutions are numerous on the string landscape, but duals are unknown (string coupling is fixed, so dual has no classical limit).

• Replacing AdS with flat spacetime or eternally inflating spacetime probably requires new concepts. So knowing string theory with AdS b.c. does not answer all important questions.

AdS/CFT duality is still unproven, as are the many string theory dualities, and even purely field-theoretic dualities like Montonen-Olive (weak-strong N = 4 SUSY Yang-Mills).

• Maldacena's `derivation': assume that the following diagram commutes:

	Weak coupling:		*Strong coupling*:
High energy:	D3-branes	↔	Black 3-branes
	↕		↕
Low energy:	N = 4 SYM	↔	$AdS_5 \times S^5$

Explained unexpected agreements between gravitational and Feynman graph calculations.

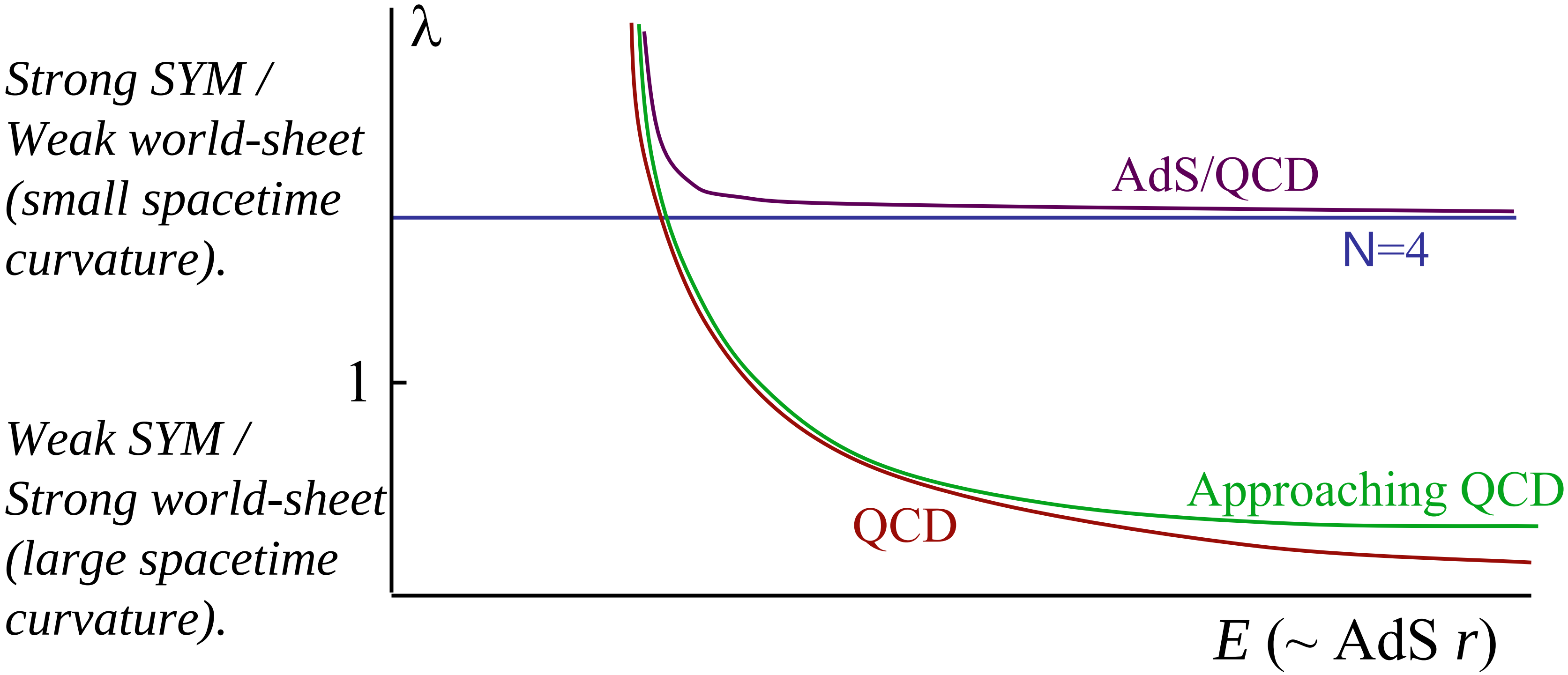
• Another derivation (Banks, Strominger): the assertion that a ten-dimensional string theory is the same a four-dimensional gauge theory is so audacious that if it were false you could disprove it in five minutes.

• Agreement in many regimes: exact SUSY, long strings, Regge, black holes (RHIC), some numerical tests (e.g. Antonuccio, Hashimoto, Lunin, Pinsky).

• String theory (*if it exists!)* in AdS_5 x S^5 defines a CFT on R^4, via the Gubser-Klebanov-Polyakov/Witten dictionary. N = 4 SYM is the only renormalizable field theory with this symmetry.

• Direct path integral manipulations (as in Ising duality) seem to need nowhere: this seems to point to the existence of deep new concepts (c.f. Riemann hypothesis). Integrability?

From CFT to QCD:

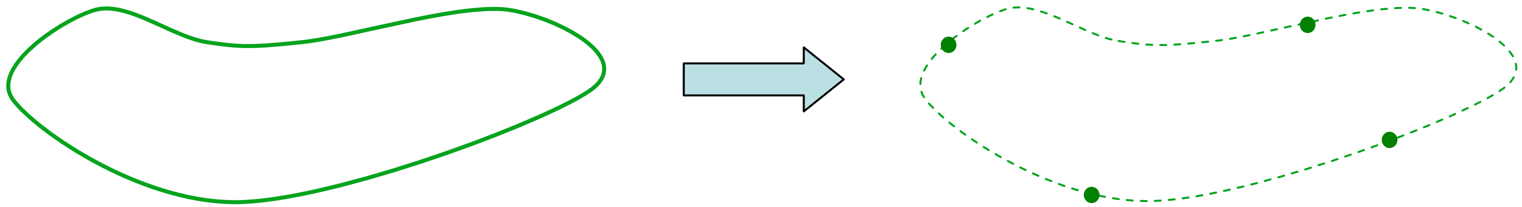


Break conf. inv.: Witten; JP+Strassler; Klebanov+Strassler

We can approach QCD as a limit: existence `proof' that large-*N* gauge theory can be written as a string theory *at strong world-sheet coupling*. 3+1 problem becomes 1+1.

To understand strongly coupled world-sheet theories study first the N = 4 theory at small and intermediate λ.

• At small λ we know that the continuous string must behave like a set of pointlike partons.



Heuristically the partons must be where the string is very close to the AdS boundary. Hard to make this quantitative (e.g. Gopakumar).

Existence proof for higher symmetries of N = 4 S-matrix?!

Can we solve a general λ? If background fields were NS-NS instead of R-R, this would be a rational CFT and solvable by standard methods. So it deserves to be exactly solvable.

• Higher charges: Bena, JP, Roiban (classical string); Berkovits (fully quantized string).

But to do with them?

• Bethe Ansatz around relativistic state $|0\rangle$ rather than charged state $|J\rangle$ (Mann and JP, in $OSp(m+2|m)$ analog model).
Advantage: world-sheet Lorentz invariance manifest.
Disadvantage: how to find S-matrix for $AdS_5 \times S^5$ theory?
Need massive integrable perturbation to find S-matrix by usual means.

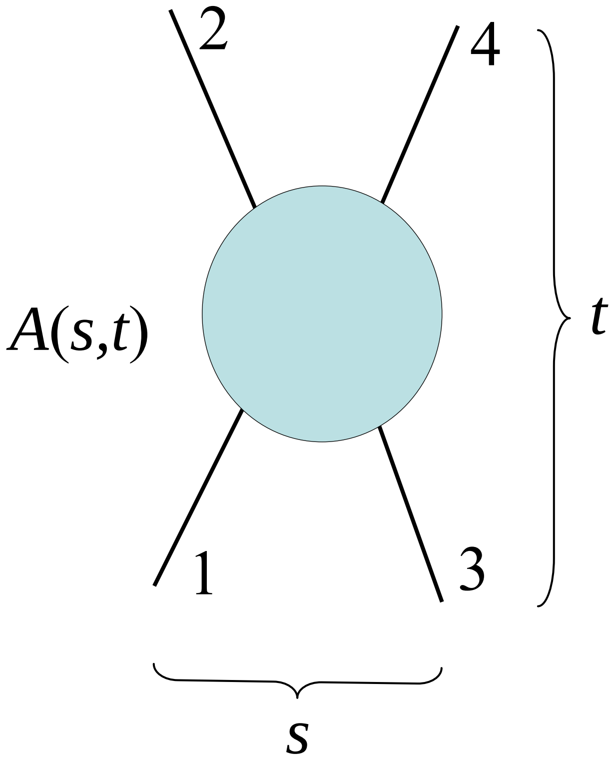
Of course, nonzero J does give mass to magnons. The Lorentz-invariance (and conformal invariance) of the underlying theory are obscured - the S-matrix no longer depends on rapidity differences - *but* it should be possible to express them as constraints on the S-matrix (cf. Janik).
Another advantage (?): very large J is like a ferromagnetic state.

The Pomeron in AdS/QCD

with R. Brower, C.-I. Tan, M. Strassler; hep-th/0603115

AdS/QCD is qualitatively similar to QCD at low energy but strongly coupled at high energy, differs from QCD in systematic ways. However, it is solvable via string dual: everything that one would want to calculate in QCD one can calculate in AdS/QCD, and this is often an instructive exercise. In particular, it gives simple insight into BFKL.

Regge Behavior



Regge limit: $s \to \infty$, t fixed.

Strong interaction and flat space string theory both show Regge behavior in Born approximation (one particle exchange):

$$A(s,t) \sim s^{\alpha(t)} .$$

Theories with a finite number of particles give

$$A(s,t) \sim s^{j} .$$

Fixed poles vs. Regge.

E.g. Virasoro-Shapiro amplitude in flat spacetime.

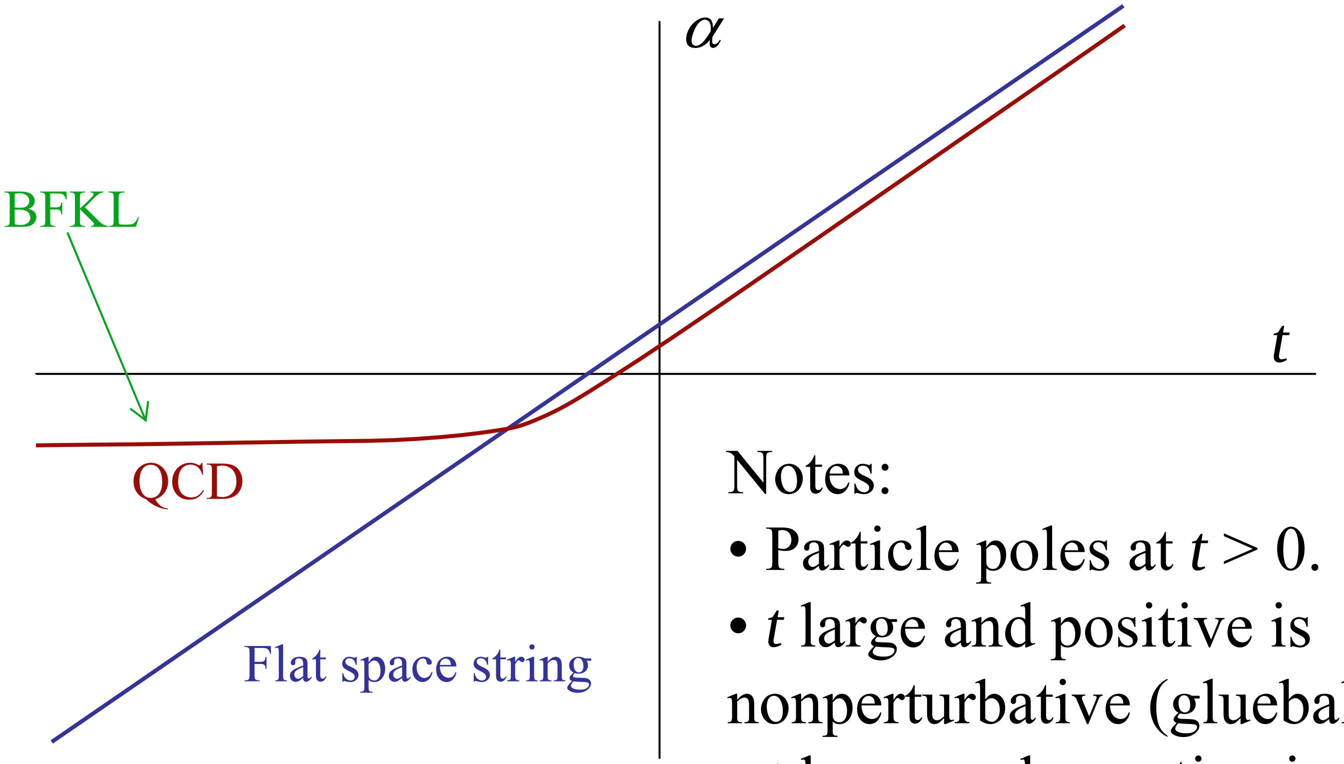
$$\frac{\Gamma(-1-\alpha's/4)\Gamma(-1-\alpha't/4)\Gamma(-1-\alpha'u/4)}{\Gamma(2+\alpha's/4)\Gamma(2+\alpha't/4)\Gamma(2+\alpha'u/4)}$$

$$\sim s^{2+\alpha't/4}(e^{-i\pi/2}\alpha')^{2+\alpha't/4}\frac{\Gamma(-1-\alpha't/4)}{\Gamma(2+\alpha't/4)}$$

Notes:

- $\alpha(t) = 2 + \alpha' t/4$ is linear in t.
- There are poles when $\alpha(t) = 2, 3, 4, \ldots$, corresponding to spins of physical particles.

Trajectories are different in QCD:



Notes:
- Particle poles at $t > 0$.
- t large and positive is nonperturbative (glueballs).
- t large and negative is perturbative (BFKL).

What can we learn from AdS/QCD?

Simpified model:

Scattering takes place in AdS_5 (reduce on S^5) with IR cutoff:

$$ds^2 = \frac{r^2}{R_{\mathrm{AdS}}^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R_{\mathrm{AdS}}^2}{r^2}dr^2 \ , \quad r > r_0 \ .$$

The AdS scale $R_{\mathrm{AdS}} \sim \lambda^{1/4} \alpha'^{1/2}$ is larger than the string length.

Due to warping, momentum $\tilde{p}^\mu$ seen by inertial observer is related to momentum p^μ seen by 4-d observer via

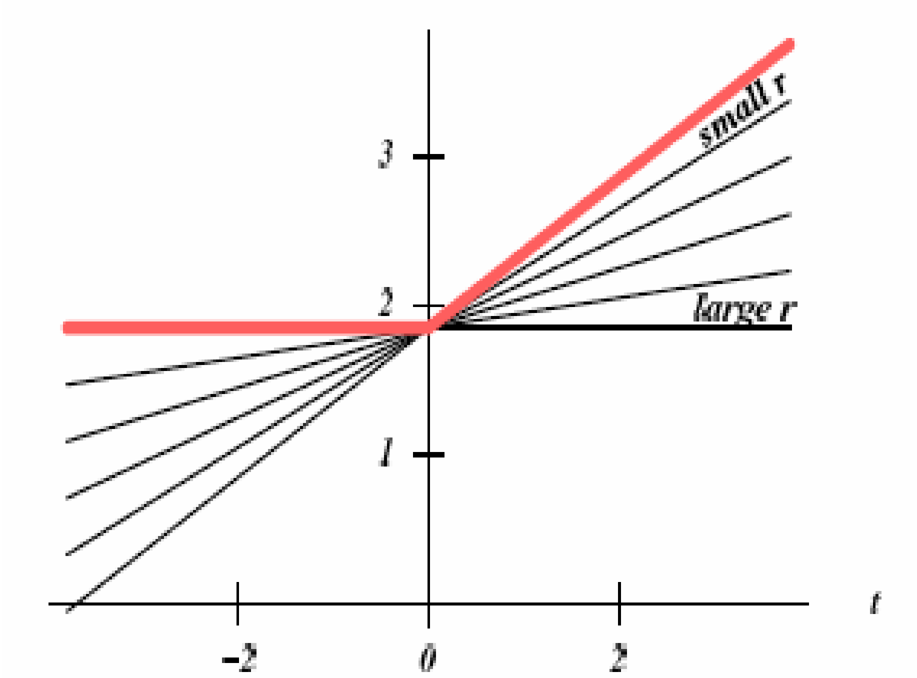
$$\tilde{p}^\mu = \frac{R_{\mathrm{AdS}}}{r}p^\mu$$

Hadrons (e.g. glueballs) are cavity modes in AdS (plane waves in x^μ times normalizable function of r). Treat scattering as approximately local in r and superpose amplitudes for scattering at different r.

The Regge exponent is

$$\alpha(t) \sim 2 + \alpha'\tilde{t}/4 = 2 + (R_{\mathrm{AdS}}/r)^2 \alpha' t/4 .$$

That is, there is an effective $\alpha'(r) = (R_{\mathrm{AdS}}/r)^2 \alpha'$; again, $r > r_0$.
Thus we see a superposition of many trajectories



Dominant contribution resembles QCD, but we can do better.

Vertex operator representation:

$$\begin{aligned} A(s,t) &= \int d^2w \, \langle V_1(0)V_2(w)V_3(1)V_4(\infty)\rangle \\ &= \int d^2w \, |w|^{-4-\alpha' t/2}|1-w|^{-4-\alpha' s/2} \\ &\sim \int d^2w \, |w|^{-4-\alpha' t/2} e^{\alpha' s(w+\bar{w})/4} \\ &= 2\pi \frac{\Gamma(-1-\alpha' t/4)}{\Gamma(2+\alpha' t/4)}(e^{-i\pi/2}\alpha' s/4)^{2+\alpha' t/2} \, . \end{aligned}$$

In Regge limit, small *w* dominates (after appropriate analytic continuations): *sw* ~ 1. Thus we should be able to make the small-*w* approximation directly at the operator level, using the OPE.

Usual OPE, w << 1:

$$e^{ip_1\cdot X(w,\bar w)}\, e^{ip_2\cdot X(0)} \sim |w|^{-4-\alpha' t/2} e^{ik\cdot X(0)}\,, \quad k = p_1 + p_2\,.$$

Regge OPE, w << 1, ws ~ 1.

$$e^{ip_1\cdot X(w,\bar w)}\, e^{ip_2\cdot X(0)} \overset{\text{Regge}}{\sim} |w|^{-4-\alpha' t/2} e^{ik\cdot X(0) + ip_1\cdot(w\partial + \bar w\bar\partial)X(0)}\,.$$

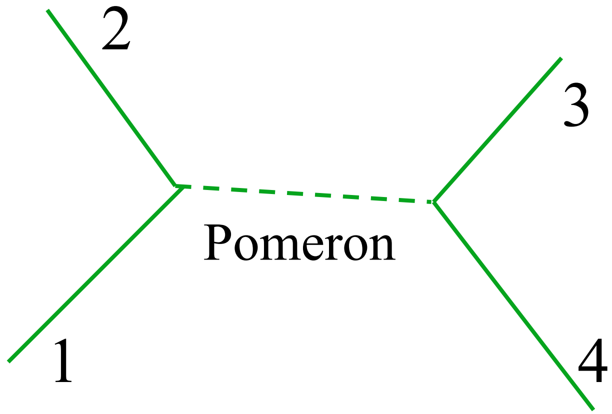
Integrate in operator form:

$$\int d^2w\, e^{ip_1\cdot X(w,\bar w)}\, e^{ip_2\cdot X(0)} \overset{\text{Regge}}{\sim} \Pi(\alpha' t) e^{ik\cdot X(0)} \big[\partial X^+(0)\bar\partial X^+(0)\big]^{1+\alpha' t/4}\,,$$

$$\Pi(\alpha' t) = 2\pi (p_1^-)^{2+\alpha' t/2} \frac{\Gamma(-1-\alpha' t/4)}{\Gamma(2+\alpha' t/4)} e^{-i\pi - i\pi\alpha' t/4}\,.$$

Here $p_1^- \sim s$ so the Regge behavior is explicit.
Point to note: fractional powers of $\partial X^+ \bar\partial X^+$.

Interpretation: amplitude factorizes on Pomeron, vertex



operator $V_P =$

$$e^{ik\cdot X(0)}\left[\partial X^+(0)\bar{\partial}X^+(0)\right]^{1+\alpha' t/4}$$

Pomeron has spin $j_P = \alpha(t) = 2 + \alpha' t/2$. It is not a physical particle: there are poles only for integer $\alpha(t)$.

The Pomeron lives in an unusual Hilbert space: the vertex operator factor $\partial X^+\bar{\partial}X^+$ maps to the string excitation $\alpha^+_{-1}\,\tilde{\alpha}^+_{-1}$, so there is fractional excitation number.

The Pomeron satisfies the physical state conditions. Its spin (and so the Regge exponent) is determined by

$$0 = L_0 - 1 = N + \alpha' k^2/4 - 1 = j/2 - \alpha' t/4 - 1\ .$$

Extend to AdS spacetime: wavefunctions, vertex operators now depend on AdS coordinate *r* as well, e.g.

$$V_{\rm P} = (\partial X^+ \bar\partial X^+)^{j/2} e^{ik\cdot X} \phi_{+j}(r)$$

Physical state condition like flat spacetime but with $-k^2$ replaced by a covariant Laplacian Δ_2*,

$$1 = L_0 = \frac{j}{2} - \frac{\alpha'}{4}\Delta_2 \cong \frac{j}{2} - \frac{1}{4\sqrt{\lambda}}\left(\frac{\partial^2}{(\partial \ln r)^2} - 4\right)^2 - \frac{\alpha'(r)t}{4}$$

Pomeron propagator $s^j = s^{\alpha'\Delta_2/4}$. Physical effects: running slope and *diffusion* with respect to ln *r*. BFKL: $s^{H_{\rm BFKL}}$ generates diffusion with respect to *size* of the two-gluon state.

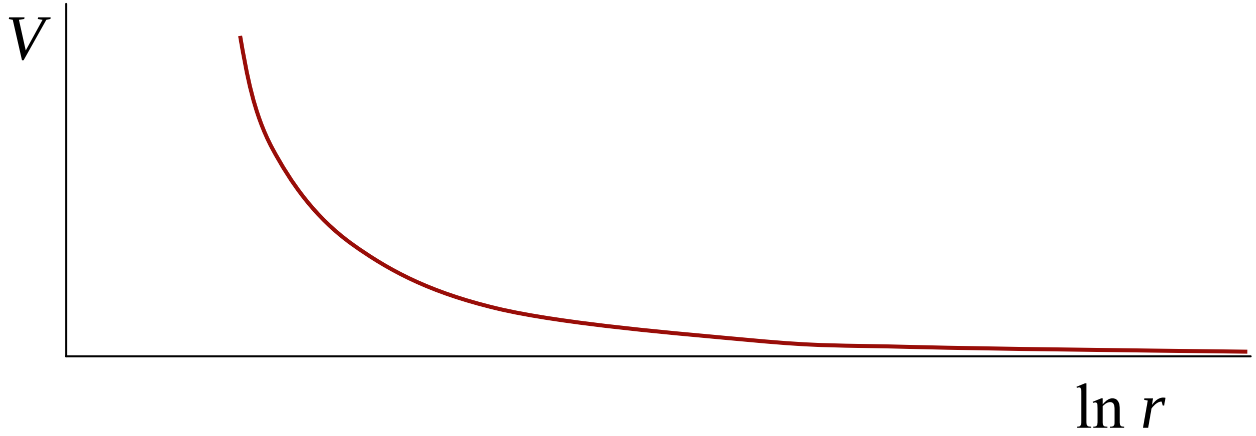
This is an important general property of AdS/CFT duality: a string state maps to a color singlet, whose size (dipole moment) is the fifth coordinate (color transparency).

*Poor notation: not the same as operator dimension Δ.

Asymptotics of Pomeron propagator $s^j = s^{\alpha' \Delta_2/2}$,

$$\frac{\alpha'}{4}\Delta_2 \cong \frac{1}{4\sqrt{\lambda}}\left(\frac{\partial^2}{(\partial \ln r)^2} - 4\right)^2 + \frac{\alpha' R_{\rm AdS}^2 t}{4r^2}$$

$-\Delta_2 \sim$ one-dimensional Schrodinger operator, potential



Lowest eigenvalue gives highest trajectory, $j = 2 - 1/(2\lambda^{1/2})$.
This is the strong-coupling value of the BFKL exponent, whose weak coupling value is $j = 1 + \lambda(\ln 2)/\pi^2$.

The BFKL operator and the anomalous dimension

In AdS/CFT, the vertex operators $r^{\kappa}(\partial X^+ \bar{\partial} X^+)^{j/2}$ map to string states, which are dual to `twist two' local operators; e.g. $j = 2$ is the graviton, which maps to $T_{\mu\nu}$.

An operator of spin j and dimension Δ gives a normalizable solution $r^{2-\Delta}$ and a non-normalizable solution r^{D-2} . The ground state of the analog potential model is r^0, corresponding to $\Delta = 2$.

The same physical state condition $L_0(j, \Delta) = 1$ determines both the dimension and the Regge exponent. For the first, fix j and solve for Δ. For the second, fix $\Delta = 2$ and solve for j.

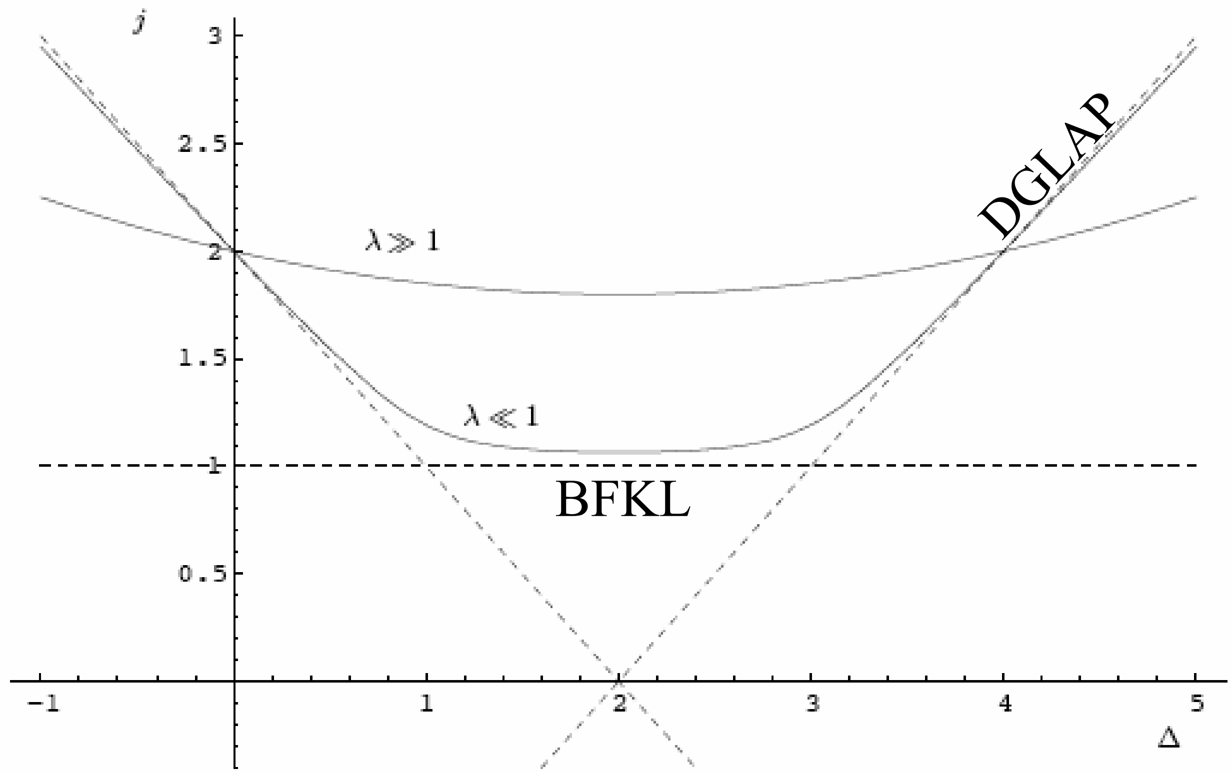
However, it's not quite that simple. At weak coupling $L_0(j,\Delta) = 1$ has at least two branches:

DGLAP (anomalous dimension): $\Delta - j - 2 = g^2 f(j)$.

BFKL: $j - 1 = g^2 h(\Delta)$.

In both cases the function on the RHS has poles, from Ψ (Digamma) function, at the point where it intersects the other branch.

Thus, the field theory Hamiltonians on the two branches are very different, though each is integrable.



At weak coupling the anomalous dimension operator and BFKL Hamiltonian represent *different* expansions, though both have integrable properties.

The BFKL calculation gives some all-orders information about the pole in the anomalous dimension at $j = 1$.

Pomerons with large R charge?

Conclusions