Strings, D-Branes and Gauge Theories

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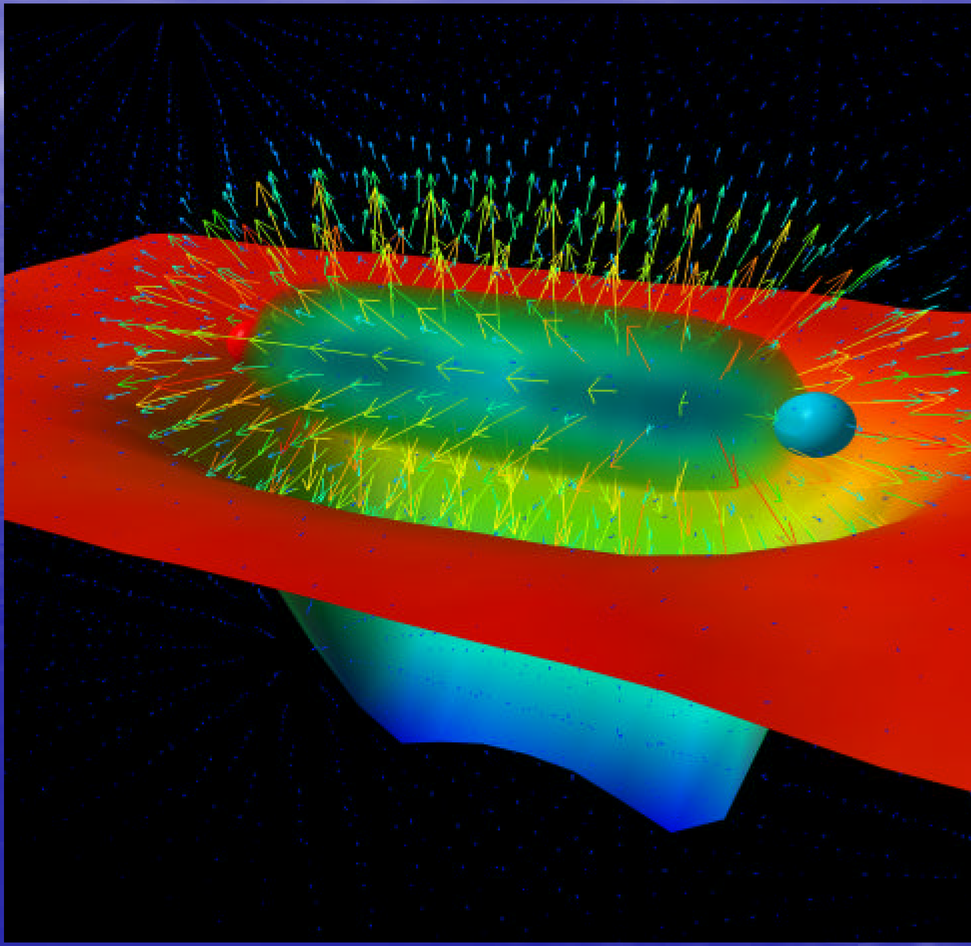
QCD and string theory

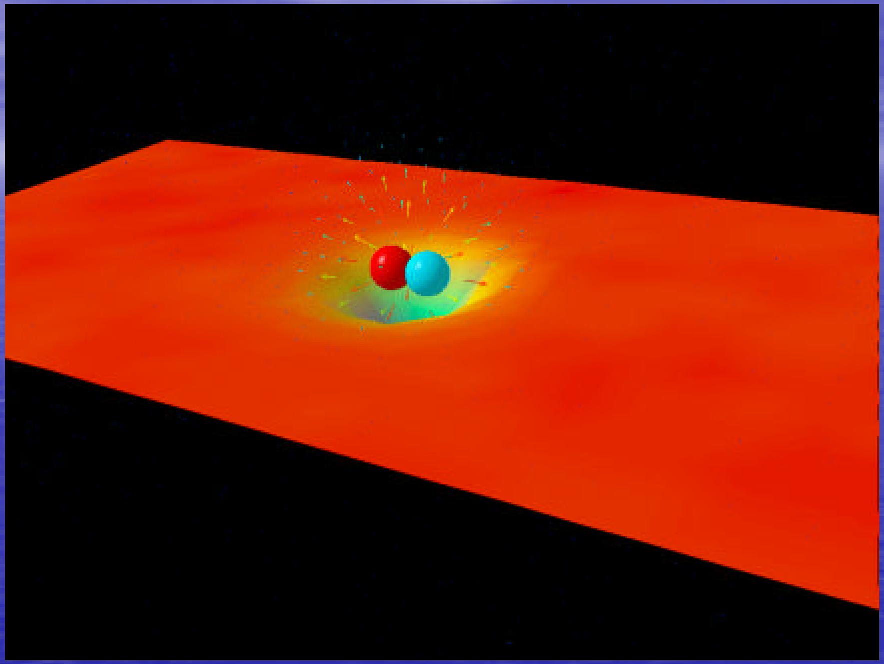
- At short distances, must smaller than 1 fermi, the quark-antiquark potential is approximately Coulombic, due to the Asymptotic Freedom.
- At large distances the potential should be linear (Wilson) due to formation of confining flux tubes.

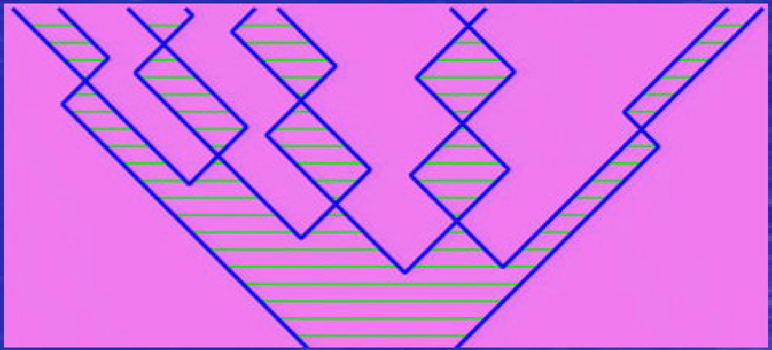


Flux Tubes in QCD

- These objects may be approximately described by the Nambu strings
  (animation from lattice work by D. Leinweber et al, Univ. of Adelaide)

- The tubes are widely used, for example, in jet hadronization algorithms (the Lund String Model) where they snap through quark-antiquark creation.

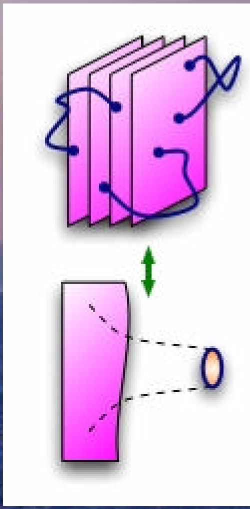




Large N Gauge Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling $\lambda = g_{\rm YM}^2 N$ fixed.
- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.

Breaking the Ice



Dirichlet branes (Polchinski) led string theory back to gauge theory in the mid-90's.

A stack of N Dirichlet 3-branes realizes *N*=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1+\frac{L^4}{r^4}\right)^{-1/2}\left(-(dx^0)^2+(dx^i)^2\right)+\left(1+\frac{L^4}{r^4}\right)^{1/2}\left(dr^2+r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

Successful matching of graviton absorption by D3-branes, related to 2-point function of stress-energy tensor in the SYM theory, with a gravity calculation in the 3-brane metric (IK; Gubser, IK, Tseytlin) was a precursor of the AdS/CFT correspondence.

The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the *N*=4 SYM theory this compact space is a 5-d sphere.
- The SO(2,4) geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The d-dimensional AdS space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 .$$

- Its metric is

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2}(dx^i)^2\right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\mathrm{YM}}^2 N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator dimension is determined by the mass of the dual field; e.g. for scalar operators

$$\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$$

- Correlation functions are calculated from the dependence of string theory path integral on boundary conditions ϕ_0 in AdS_5, imposed near z=0:

$$\langle \exp \int d^4x \phi_0 \mathcal{O} \rangle = Z_{\rm string}[\phi_0]$$

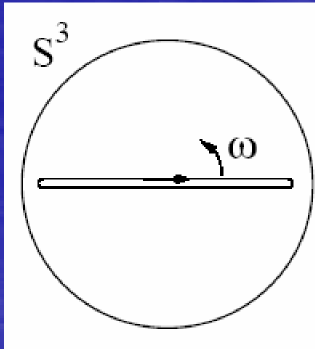
- In the large N limit the path integral is found from the classical string action: $Z_{\rm string}[\phi_0] \sim \exp(-I[\phi_0])$

Spinning Strings vs. Long Operators

- Vibrating closed strings with large angular momentum on the 5-sphere are dual to SYM operators with large R-charge. Berenstein, Maldacena, Nastase
- Generally, semi-classical spinning strings are dual to long operators, e.g. the dual of a high spin operator

$$\mathrm{Tr}\, F_{+\mu} D_+^{J-2} F_+{}^{\mu}$$

is a folded string spinning around the center of AdS_5. Gubser, IK, Polyakov



- The anomalous dimension of such a high spin twist-2 operator is $\Delta - (J+2) \to f(\lambda)\ln J$
- AdS/CFT predicts that at strong coupling $f(\lambda) \to \frac{\sqrt{\lambda}}{\pi}$
- A 3-loop perturbative *N*=4 SYM calculation gives
Kotikov, Lipatov, Onishchenko, Velizhanin; Bern, Dixon, Smirnov

$$f(\lambda) = \frac{1}{2\pi^2}\left(\lambda - \frac{\lambda^2}{48} + \frac{11\lambda^3}{11520} + O(\lambda^4)\right)$$

- An approximate extrapolation formula works with about 10% accuracy:

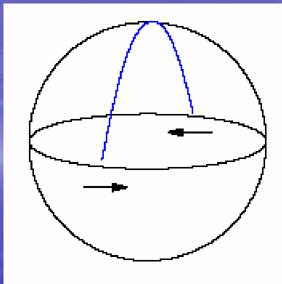
$$\tilde{f}(\lambda) = \frac{12}{\pi^2}\left(-1 + \sqrt{1+\lambda/12}\right) = \frac{1}{2\pi^2}\left(\lambda - \frac{\lambda^2}{48} + \frac{\lambda^3}{1152} + O(\lambda^4)\right)$$

- Calculation of f(λ) was recently addressed using integrability by Eden and Staudacher; Belitsky et al

Folded Strings on S^5 and Magnons

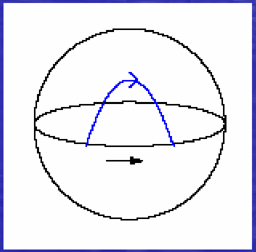
- Folded string spinning around the north pole has, for large R-charge J: $E - J = 2\frac{\sqrt{\lambda}}{\pi}$

Gubser, IK, Polyakov



- Recently this string was identified by Hofman and Maldacena with a 2-magnon operator $\mathrm{Tr}\, W Z^l W Z^{J-l} e^{ipl}$ where each magnon carries p=π . The exact energy of a free magnon $\sqrt{1+\frac{\lambda}{\pi^2}\sin^2(p/2)}$ is BPS protected

Beisert



- Its dual is an open string spinning on S^5 :
- Hence the 2-magnon operator dimension is

$$\Delta = J + 2\sqrt{1+\frac{\lambda}{\pi^2}} + O(1/J)$$

in exact agreement with the folded string result for large λ.

- The magnon energy as a function of ● exhibits the general features expected of a sum over planar graphs.
- A finite radius of convergence.
- A branch point at a negative value of λ which does not prevent a smooth extrapolation from 0 to infinity along the positive real axis (a flat direction of the *N*=4 SYM).

- Another (probably) exact result with similar features is the expectation value of a circular Wilson loop obtained from rainbow graph summation Erickson, Semenoff, Zarembo

$$\langle W \rangle_{\text{rainbow}} = \frac{2}{\sqrt{\lambda}} I_1\left(\sqrt{\lambda}\right)$$

- Here the radius of convergence of ln W is related to the zero of the Bessel function J_1.
- The strong coupling limit

$$\langle W \rangle_{\text{rainbow}} \sim \sqrt{\frac{2}{\pi}} \frac{e^{\sqrt{\lambda}}}{\lambda^{3/4}}$$

is in agreement with the AdS calculation.

Entropy of thermal supersymmetric SU(N) theory

- Thermal CFT is described by a black hole in AdS_5

$$ds^2_{BH} = \frac{L^2}{z^2}\left(\frac{dz^2}{1 - z^4/z_h^4} - (1 - z^4/z_h^4)(dx^0)^2 + \sum_{i=1}^{3}(dx^i)^2\right)$$

- The CFT temperature is identified with the Hawking T of the horizon located at z_h
- Any event horizon contains Bekenstein-Hawking entropy $S_{BH} = \frac{2\pi A_h}{\kappa^2}$
- A brief calculation gives the entropy density $s = \frac{\pi^2}{2} N^2 T^3$

Gubser, IK, Peet

- This is interpreted as the strong coupling limit of

$$s = \frac{2\pi^2}{3} f(\lambda) N^2 T^3$$

- For small `t Hooft coupling, Feynman graph calculations in the N =4 SYM theory give

$$f(\lambda) = 1 - \frac{3}{2\pi^2}\lambda + \frac{3+\sqrt{2}}{\pi^3}\lambda^{3/2} + \ldots$$

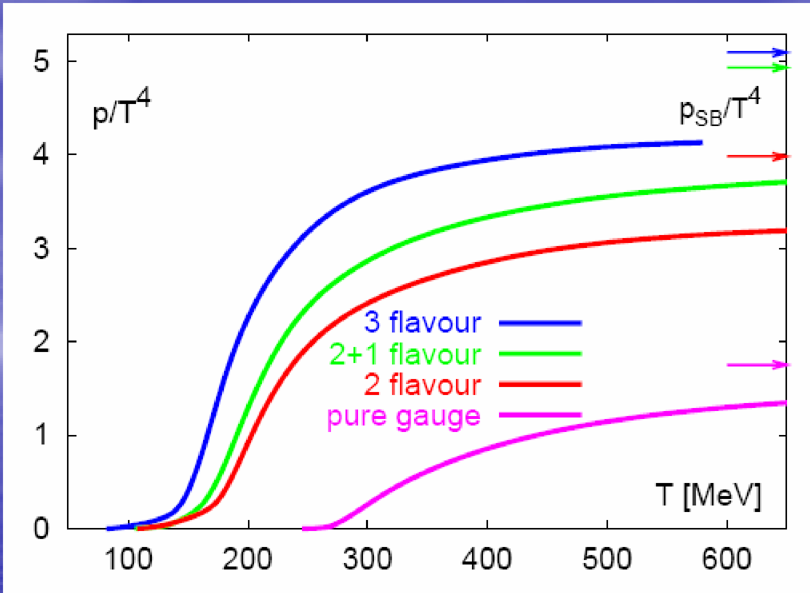
- We deduce from AdS/CFT duality that

$$\lim_{\lambda\to\infty} f(\lambda) = \frac{3}{4}$$

- The entropy density is multiplied only by ¾ as the coupling changes from zero to infinity. Gubser, IK, Tseytlin

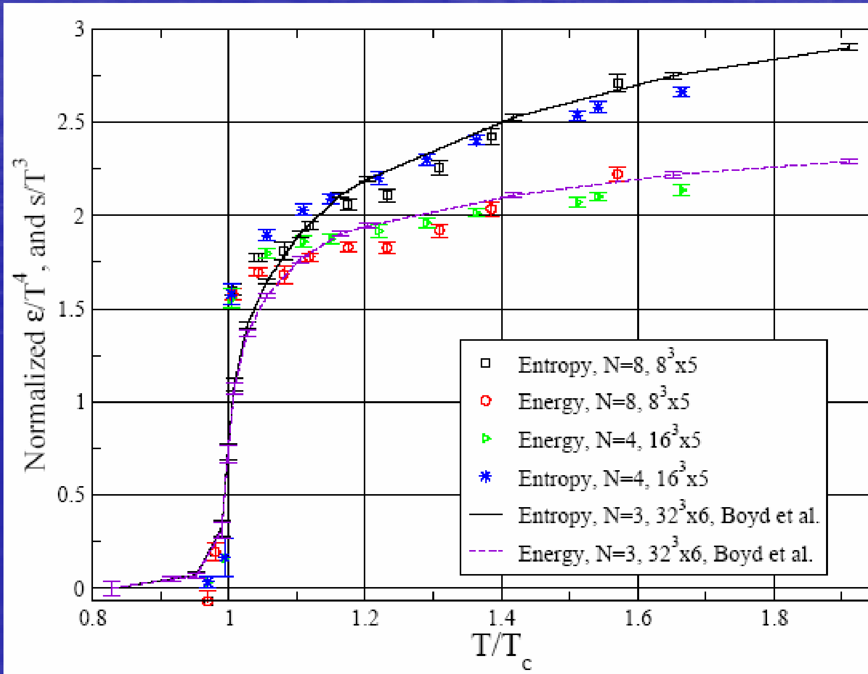
- A similar effect is observed in lattice simulations of non-supersymmetric gauge theories for N=3: the arrows show free field values.

  Karsch (hep-lat/0106019).



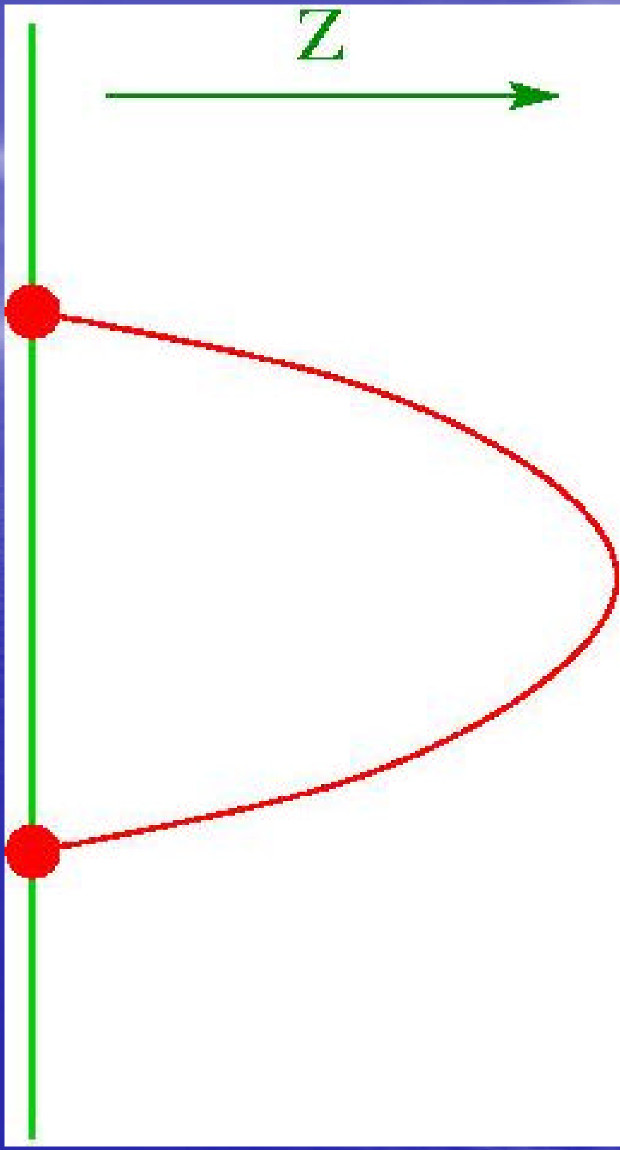
- N-dependence in the pure glue theory enters largely through the overall normalization.

  Bringoltz and Teper (hep-lat/0506034)



Quark Anti-Quark Potential

- The z-direction is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.
- In a pleasant surprise, because of the 5-th dimension z, the string picture applies even to theories that are conformal (not confining!). The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential (Maldacena; Rey, Yee)



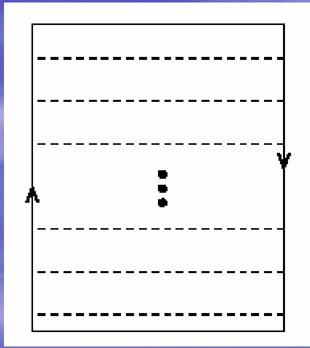
$$E_0(L) = -c\frac{\sqrt{g^2 N}}{L}$$

Excitations of the Gluonic String

Callan, Guijosa; Brower, Tan, Thorn; IK, Maldacena, Thorn

- The spectrum of small oscillations of the string with Dirichlet boundary conditions at z=0 is known: $E_{\{N_n\}} - E_0 = \sum N_n \omega_n$
- By conformal invariance, all energies scale as 1/L. Their spectrum is known, and they correspond to gluonic excitations at strong `t Hooft coupling.
- But at weak coupling there are no such excitations: only the ground state of energy $-\frac{\pi\hat{\lambda}}{2L}$ where $\hat{\lambda} = g^2 N/4\pi^2$

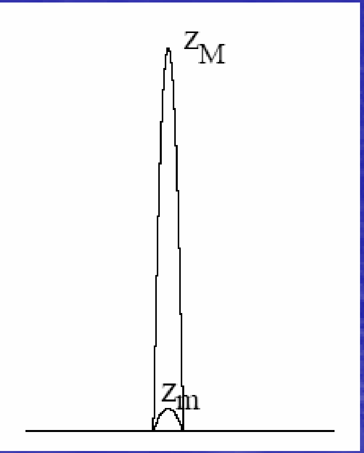
- A simplified model where only ladder graphs are summed indicates that the coupling where an infinite number of excitations appear is $\hat{\lambda}_c = 1/4$



- Their appearance is related to the fall-to-the center instability in the bound state equation

$$\left[-\partial_t^2 - \frac{\hat{\lambda}}{L^2+t^2}\right]\psi = -\frac{E^2}{4}\psi$$

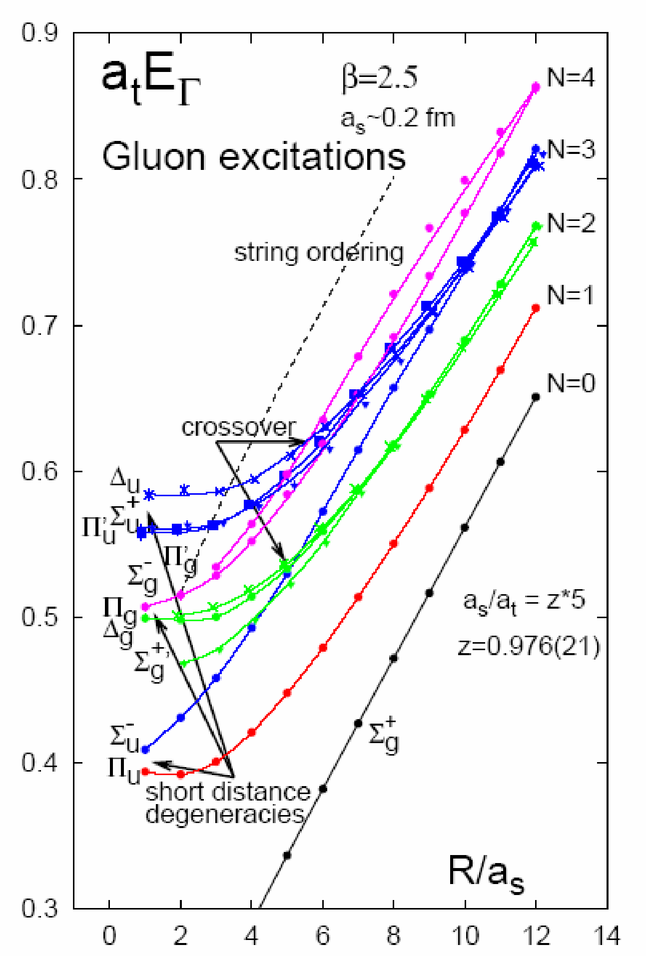
- The near-threshold bound states are in exact agreement with the spectrum of a highly excited string containing a single very long fold:



$$-\log(-Ez_m) \sim \frac{\pi^2\alpha'}{R^2}n \sim \frac{\pi^2}{\sqrt{g^2N}}n$$

Is such a `fall to the center' transition possible in QCD?

- Apparently not, since the asymptotic freedom makes the coupling weak when the ends of the string approach each other.
- A recent lattice calculation of the string excitation spectrum shows this explicitly. Only one level exists with energy much lower than others at short separation. Juge, Kuti, Morningstar



Why are the flux tube excitations stable in a CFT ?

- For a combination of energetic and large N reasons!
- After a gluon is emitted the resulting state would be in a color adjoint, and would have positive energy. Hence, energy conservation forbids one-gluon emission from an excited bound state.
- The energy conservation does not forbid glueball (closed string) emission. But it is suppressed at large N.

Interpolating functions

- Interpolating functions of `t Hooft coupling are a general feature of planar CFT.
- Such functions can easily explain the lack of agreement between spinning string (strong coupling) predictions and perturbative gauge theory results at 3 loops:

$$E/J = a_0 + a_1\, \lambda/J^2 + a_2\, \lambda^2/J^4 + a_3(\lambda)\, \lambda^3/J^6 + \ldots$$

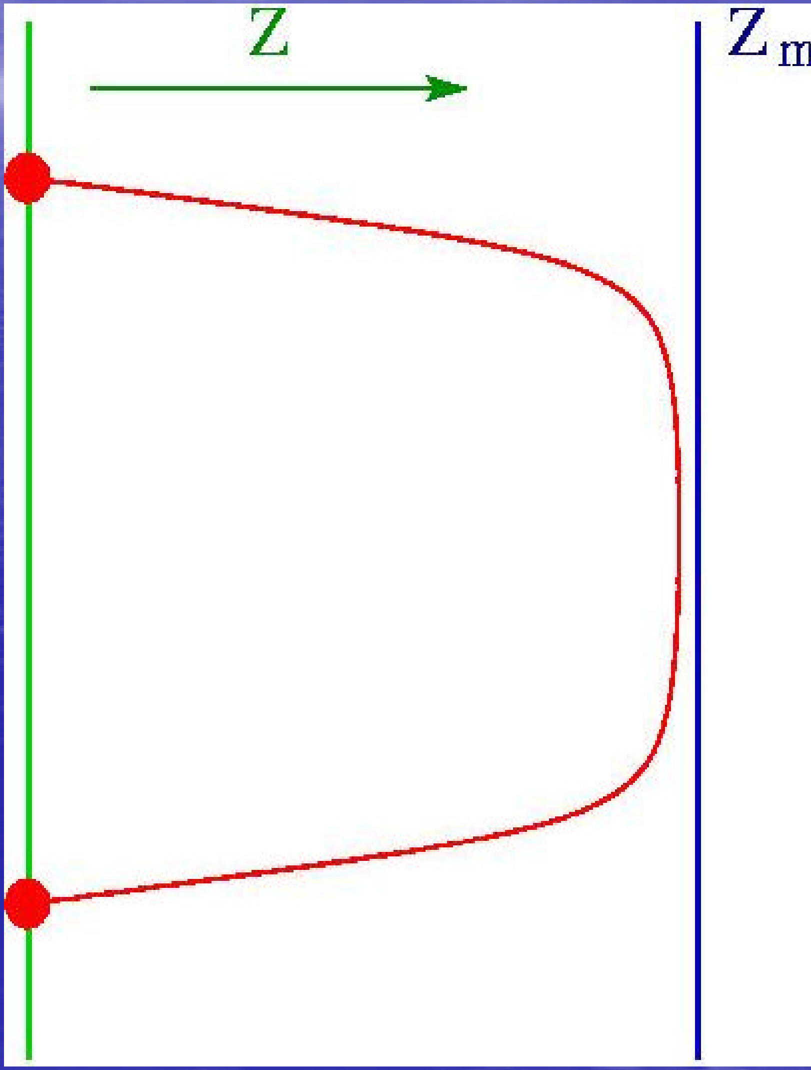
- This would imply a perturbative `BMN scaling' violation appearing at 4 loops or higher.
- Interpolating functions are expected to have a finite radius of convergence.
- In most cases the singularities should lie away from the positive real axis, but some special quantitites may have singularities there and exhibit real phase transitions.

String Theoretic Approach to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- A “cartoon” of the necessary metric is

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\left(-(dx^0)^2 + (dx^i)^2 \right)$$

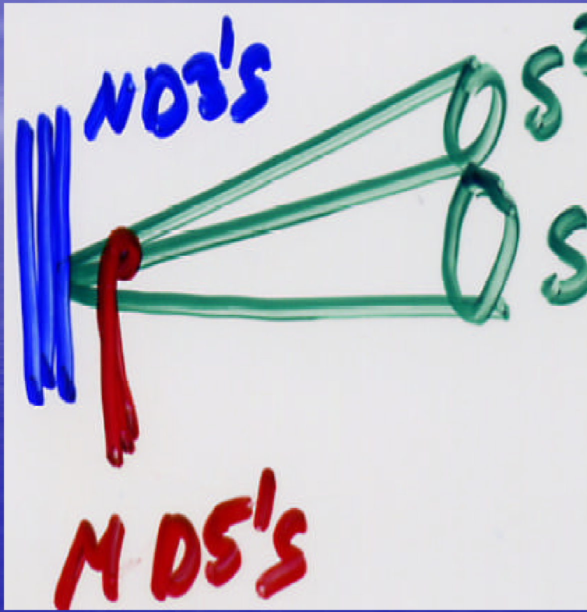
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\max})}{2\pi\alpha'}$



- Several 10-dimensional backgrounds with these qualitative properties are known (the compact space is actually “mixed” with the 5-d space).
- Witten (1998) constructed a background in the universality class of non-supersymmetric pure glue gauge theory. While there is no asymptotic freedom in the UV, hence no dimensional transmutation, the background serves as a simple model of confinement.
- Many infrared observables may be calculated from this background using classical supergravity. The lightest glueball masses are found from normalizable fluctuations around the supergravity solution. Their spectrum is discrete, and resembles qualitatively the results of lattice simulations in the pure glue theory.

Confinement in SYM theories

- Introduction of minimal supersymmetry ($\mathcal{N}$=1) facilitates construction of gauge/string dualities.
- A useful tool is to place D3-branes and wrapped D5-branes at the tip of a 6-d cone, e.g. the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)



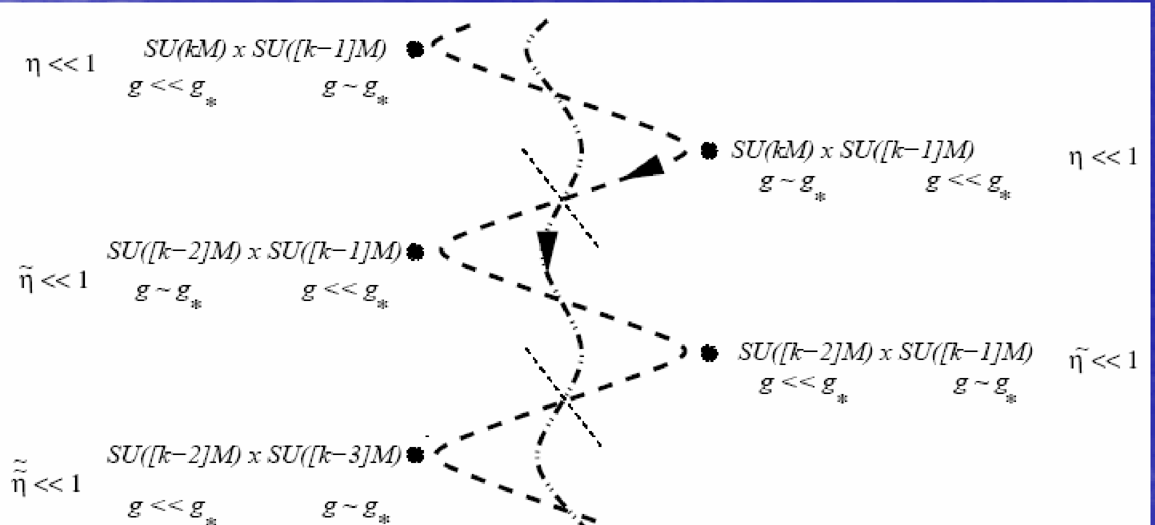
$$ds^2_{10} = h^{-1/2}(\tau)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(\tau)ds^2_6$$

- ds^2_6 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- In the UV there is a logarithmic running of the gauge couplings. Surprisingly, the 5-form flux, dual to N, also changes logarithmically with the RG scale. IK, Tseytlin
- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a duality cascade, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler

(diagram of RG flows from a review by M. Strassler)



- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: Z_{2M} -> Z_{2}.
- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f=N_c$.
- The baryon and anti-baryon operators Seiberg

$$\mathcal{A} = \epsilon^{i_1 \ldots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \ldots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$

$$\mathcal{B} = \epsilon_{i_1 \ldots i_{N_c}} B^{i_1}_{\dot{\alpha}_1 a_1} \ldots B^{i_{N_c}}_{\dot{\alpha}_{N_c} a_{N_c}}$$

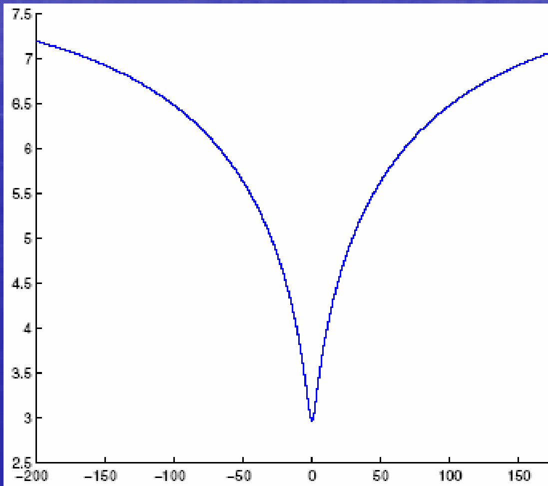
acquire expectation values and break the U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner.

- The KS solution is part of a moduli space of confining SUGRA backgrounds, resolved warped deformed conifolds.

Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni

- This family of solutions is dual to the `baryonic branch' in the gauge theory:

$$\mathcal{A} = i\Lambda_1^{2M}\zeta \ , \qquad \mathcal{B} = i\Lambda_1^{2M}/\zeta$$



- Using the SUGRA solutions, various quantities have been calculated as a function of the modulus U=ln |ζ|.
- Here is a plot of the string tension: a fundamental string at the bottom of resolved warped deformed conifold is dual to an `emergent' chromo-electric flux tube.

Dymarsky, IK, Seiberg

- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for testing various ideas.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov (work in progress)
- Are there any integrable confining 4-d gauge theories?

Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions
- The AdS/CFT correspondence makes this connection precise. It makes a multitude of dynamical statements about strongly coupled conformal (non-confining) gauge theories. Integrability in AdS/CFT should be important for understanding interpolation from small to large ●.
- Gauge/string dualities for confining theories provide a new geometrical view of such important phenomena as dimensional transmutation and chiral symmetry breaking. This allows for calculations of glueball and meson spectra. Are there integrable confining gauge theories?